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Regional competitiveness of selected Sub-Saharan African economies – an application of stochastic production frontier analysis

This article evaluates the competitiveness of 44 selected Sub-Saharan African economies by modelling the efficient utilization of the factors of production. It deviates from the traditional approach and methods for a competitiveness study and opts to utilize the econometric methodology of stochastic production frontier, using Cobb-Douglas production function to estimate time-invariant and time-varying decay effects efficiency and panel data for 1980–2019. The results show that the selected SSA countries operated on an average score of 40% and 26% efficiency levels, when analyzing the data under time-invariant and time-varying decay models respectively. Highly competitive countries ranked higher with respect to efficiency, incl. Equatorial Guinea, Mauritius, South Africa, Eswatini, and Gabon. At the bottom of the scale were Congo, Liberia, Burundi, Central Africa, and Niger.

Keywords: technical efficiency, stochastic production frontier, Cobb-Douglas production function, time-varying decay model, truncated normal distribution

JEL classification: E23, H21, O55, P52

Introduction

Sub-Saharan Africa is a diverse economic region with abundant human and natural resources that can economically progress to improve the standard of living of its populace. More than 1 billion people live in the SSA region, with those under the age of 25 estimated to constitute half of the total population by 2050 [WB]. However, SSA countries are poor due to their underperforming economies, with high corruption, poor infrastructure and weak and inefficient public institutions causing lower productivity growth and impeding economic progress.

Figure 1 depicts GNI per capita in constant 2017 USD based on PPP of the world's developing regions in 2000–2018. It compares one of the key stylized facts of economic growth and improvement in the income level per capita of the SSA region with the rest of the world. The GNI per capita in SSA remains stagnant,

which is a departure from the upwards growth trend of the major economic regions. The implications for the SSA countries are lower GDP and economic growth rates, which translate into higher poverty rates.

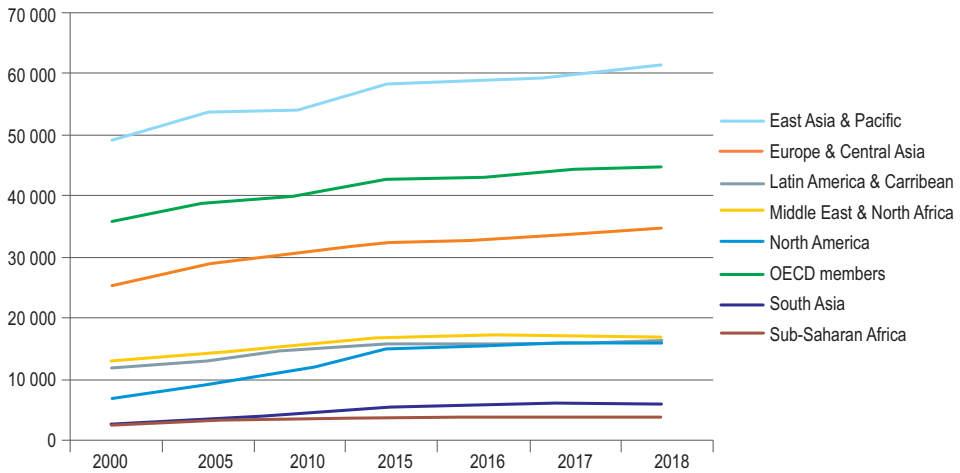


Figure 1. GNI per capita across world developing regions in 2000–2018 (constant 2017 USD, PPP)

Source: [WB].

Significantly, there have been poverty rates improvements globally, but SSA countries still have the highest poverty rates in the world. According to the World Population Review (poverty rate by country), which is based on the 2021 World Bank estimates, ca. 736 million people live in extreme poverty, surviving on less than USD 1.90 per day, and out of this number, it is estimated that 413 million live in Sub-Saharan Africa.

Many questions remain unanswered as to the cause of the lower economic growth and development within SSA. Therefore, the important task now is to evaluate the salient factors that determine the economic growth of SSA countries.

1. Objectives

This paper aims to analyze the competitiveness of SSA countries in their quest for economic growth and development. Many of them had recently enjoyed economic success, e.g. Rwanda, Ghana, Kenya, Uganda, Ivory Coast, and Ethiopia have ambitious plans to grow and develop and transform into upper-middle-income economies. The essence is an assessment of these countries' competitive

use of their available resources to ensure economic growth that will transform their economies for the betterments of their citizenries. In short, this paper empirically evaluates the competitiveness of SSA countries based on the available factors of production, using the scale of efficiency. It deviates from the previous competitiveness studies, most of which are conducted and centered in advanced countries like Japan, the UK, Canada, Poland, Ireland, Germany, or Italy, at the disadvantage and neglect of developing countries, particularly within SSA. Secondly, because of the vast differences in economic structures between advanced and developing countries, the application of their findings may not be transferable. The need to evaluate the competitiveness of SSA countries in particular is therefore warranted.

2. Methodology and theoretical framework

Over the past few years, assessments of economic policies, growth, and development have gained recognition in attempts to compare the competitiveness of countries and sub-regions. The economic success and competitiveness of countries directly depict the efficiency of the factors of production. Over a decade ago, the importance of efficiency in economic growth and development was emphasized. There is a consensus that economic efficiency positively impacts economic growth and development. In other words, how effectively the resources of a country are combined and utilized significantly expedites that country's economic progress, which also improves its competitiveness.

The OECD [1994] defines competitiveness of a country as "the degree to which [it] can, under free and fair market conditions, produce goods and services which meet the requirements of international markets, while simultaneously maintaining and expanding the real incomes of its people over the long term". Competitiveness can be evaluated with the use of various methods. This means that there is no unique methodology for competitiveness assessment. This paper makes use of technical efficiency to evaluate the competitiveness of SSA countries.

Based on the fundamental theory of production, the growth of output is attributable to technical and economic efficiency, economies of scale in production, and specialization that tend to reduce costs and increase productivity. A country able to efficiently combine its factors of production in the production of goods and services tends to perform better competitively when compared with other countries [Fuente-Mella et al., 2020].

The concept of efficiency was first introduced in the 19th century by Pareto in his production and resource utilization analysis. After that, studies in efficiency were carried out in the 1950s by Koopmans and Debreu [Ouattara, 2012], but it

was after the seminar work of Farrell [1957] that the concept gained momentum. The papers that add to Farrell's efficiency measures are discussed by Hjalmarsson [1978]. Efficiency can be technical or allocative [Farrell, 1957]. Their combination constitutes economic efficiency [Battese, Coelli, 1992; 1995; Battese, 1992; Coelli et al., 2005; Singh et al., 2000].

3. Measuring efficiency

Efficiency can be measured or estimated using data envelopment analysis, a method both deterministic and nonparametric, and the stochastic frontier method, a parametric method allowing for random shocks in its estimation [Fuente-Mella et al., 2020]. Typically, analysts measure efficiency with the use of a production function, which depicts the maximum output a firm or a country can produce given the available factors of production under the existing technology [Battese, Coelli, 1992].

This paper adopts the estimation method of stochastic frontier analysis (SFA) by using the Cobb–Douglas production framework. The stochastic frontier model can be formulated as [Mango et al., 2015]:

$$Y_{it} = f(X_{it}, \beta) + v_{it} - u_{it} = f(X_{it}, \beta) + \varepsilon_{it} \quad [1]$$

where:

Y_{it} – scalar output of country i in time T ,

X_{it} – vector of factors of production in time T ,

β – vector of parameters to be estimated,

ε_{it} – composed error term that measures the level of efficiency of country i at time T .

The error term ε_{it} breaks into two parts: v_{it} , which is defined as the effects of random shocks, which is beyond the control of country i and is assumed to be independently and identically distributed (*iid*), symmetric, and distributed independently from u_{it} ; u_{it} are the nonnegative ($u_i \geq 0$) technical inefficiency effects, representing the economic and other factors under the control of country i . Therefore, the error term u_{it} captures the technical inefficiency component of the error term ε_{it} . The stochastic production frontier model is also based on the assumption that economic agent i , be it an individual, a firm, or a country, exploits the full or complete technological potential when the value of u_i comes close to zero [Mburu et al., 2014]. Thus, the higher the value of u_i , the higher the level of technical inefficiency. Because ($u_i \geq 0$), its subtraction from model 1 implies that $0 < \varepsilon_{it} \leq 1$, the assumption being that the combination of u_i and v_{it} should be between 0 and 1.

Kumbhakar and Lovell [2000] provided a detailed version of this derivation¹, and presented a similar approach in cost derivation [Sugarhouse, 2000].

This paper utilizes the unbalanced panel data from the Penn World Table, which is a set of national-accounts data developed and maintained by scholars at the University of California and the Groningen Growth and Development Centre at the University of Groningen to measure real GDP and other variables across countries and over time. The PWT panel data for 1980–2019 is considered this study [Feenstra et al., 2015].

Following Battese and Coelli [1992], the time-varying stochastic production frontier model in the Cobb–Douglas production function framework in logarithm form for technical inefficiency could be specified as:

$$\ln(RGDPO_{it}) = \alpha_i + \beta_1 \ln(EMP_{it}) + \beta_2 \ln(Year_{it}) + (v_{it} - u_{it}) \quad [2]$$

where:

$$u_{it} = \exp[-\eta(t - T_i)]u_i \quad [3]$$

$\ln(RGDPO_{it})$ – log of output-side real GDP at chained PPPs (in million 2017 USD)
 i , for period t ,

$\ln(EMP_{it})$ – number of persons engaged (in millions) in country i , for period t ,

$\ln(CN_{it})$ – capital stock at current PPPs (in million 2017 USD) of country i , for period t ,

$Year_{it}$ – trend variable, which is a proxy for technological progress,

T_i – the last period in the i^{th} panel, η = is the decay parameter,

α_i – the country i specific constant term,

v_{it} – two-sided random error component beyond the control of the country i , for period t .

u_{it} – one-sided inefficiency component.

The combination of v_{it} and u_{it} gives ε_{it} in (1) and $i = 1, \dots, N, t = 1, \dots, T$.

The econometric model in equation 3 assumes that the efficiency of each country within the sub-region might have changed over the time period 1980–2019, since there has been a structural and institutional transformation, leading to economic progress in most of the countries, and thus potential efficiency gains.

It should be noted that without model 3, the equation reduces to model 2, which is the time-invariant model at the base level, as described by Battese and Coelli [1988].

Model 2 is estimated assuming that the economies of these countries are diverse due to the differences in economic structures, the factors affecting their economies, and the way they are efficiently combining their factors of production, leading to variations of efficiency. True fixed effects (TFE) are assumed since each

¹ Analogous derivation in the dual cost function was used in the process.

country may have time-invariant characteristics such as language, culture and political system that can influence predictor variables. In this case, heterogeneity means that $\alpha = \alpha_i$ and time-varying country inefficiency u_i are considered [Rashid-ghalam et al., 2016]. Model 2 is also estimated under the assumption of maximum likelihood and under the assumption that one-sided inefficiency u_{it} has truncated normal distribution with v_{it} having a normal distribution with a mean and a standard deviation of $(0, 1)$. Thus, using maximum likelihood requires that the parametric assumptions of the error terms v_{it} and u_{it} should be $v_{it} \sim iid N(0, \sigma_v^2)$ and $u_{it} \sim iid N^+(0, \sigma_u^2)$ under truncated normal distribution. The error terms v_{it} and u_{it} are also distributed independently of each other and the covariates in model 2.

Model 2 could also be estimated under other distributions [Newton et al., 2010; Ahmadzai, 2017] and gamma distributions [Kumbhakar et al., 2015].

As proposed by Battese and Coelli [1992], the output-oriented technical efficiency scores can be predicted after estimating model 2, using the conditional expectation predictor:

$$TE_i = \exp(-u_i) = \frac{y_i}{\exp(x_i, \beta) + v_i} = \frac{y_i}{\bar{y}} \quad [4]$$

Efficiency scores are useful for assessing policy implications, and there is a need to investigate factors that cause inefficiencies [Jones, Mygind, 2008]. Inefficiency can be affected by the time trend, and we incorporate T as the time-varying inefficiency variable [Battese, Coelli, 1992]. In time-decaying specification, u_{it} is stipulated in model 3 as [Sugarhouse, 2000; Baten et al., 2009]:

$$u_{it} = \exp(\eta[t - T_i])u_i \quad [5]$$

where:

η – unknown scalar parameter to be estimated, which determines whether inefficiencies are time-varying or time-invariant.

When $\eta > 0$, the degree of inefficiency decays over time; when $\eta < 0$, the degree of inefficiency shifts upwards over time. Because $t = T_i$ in the last period, the last period for country would contain the base level of inefficiency for that country. If $\eta > 0$, the level of inefficiency reduces toward the base level. If $\eta < 0$, the level of inefficiency increases to the base level [Baten et al., 2009; Sugarhouse, 2000]. Models 2 and 3 are estimated simultaneously to avoid possible downward biased [Ahmadzai, 2017; Kumbhakar et al., 2015]. The frontier parameters to be estimated are β_1, β_2 and β_3 . The frontier estimates or output also brings out the reports for the following items: $(\sigma_v^2, \sigma_u^2; \sigma_s^2 = (\sigma_v^2 + \sigma_u^2), \gamma = (\sigma_u^2 / \sigma_s^2), \lambda = \sigma_u / \sigma_v)$ and η (time decaying parameter).

4. Results and discussion

Table 1 presents the summary statistics of the variables under study, which show significant differences. Their means and the standard deviations vary, indicating a statistical difference.

Table 1. Statistical summary of output and input variables

Variable	Mean	SD	Min.	Max.	Observation
logrgdp	9.6216	1.4034	5.8883	13.8297	1,472
logemp	0.6984	1.5484	-3.3749	4.2907	1,472
logcn	10.4514	1.6372	5.9952	15.3396	1,472
year	-	-	1980	2019	39 years

Source: Own elaboration.

In the selected SSA countries, the mean value 0.6984 of the log of the number of persons engaged (in millions), which is represented by $\log emp$, is the lowest with a minimum value of -3.3749 and a maximum value of 4.2907, compared with the rest of the means of the other variables. The log of output-side real GDP at chained PPPs (in million 2017 USD) has a mean of 9.6216, a standard deviation of 1.4034, and maximum and minimum values of 13.8297 and 5.8883, respectively, indicating that there are variations of output-side real GDP among the selected SSA countries. The standard deviation of the log of capital stock at current PPPs (in million 2017 USD), represented by $\log cn$, is the highest with the value of 1.6372 and the minimum and maximum years of 5.9952 and 15.3396, respectively, indicating how comprehensive the series of this variable is.

Table 2 presents the results of the SFA as defined in model 2. The results of Cobb–Douglas stochastic production frontier of efficiency analysis of the 44 selected SSA countries are discussed or undermentioned. The results are obtained using Stata 11.

The coefficients of employment, capital, and technological progress are significantly different from zero at 1% for truncated normal distribution in both models. Employment and capital have the expected signs, indicating that these inputs significantly impact the economic progress of the selected SSA countries. These variables promote the countries' GDP, making them more competitive. Technology, measured in the model by the trend variable (year), has an unexpected negative sign in both models, indicating technological regression, delayed economic progress, and lower competitiveness. In fact, there has been no technological development, R&D, or innovation in SSA countries. "Innovation drives

that process, it underlies economic growth, and it is a crucial element in how countries achieve prosperity" [Schumpeter, 1942].

Table 2. Maximum likelihood estimates of technical efficiency for SSA countries

Variable	Parameter	Time-invariant model	Time-varying decay model
logemp	β_1	0.689*** (0.0946)	0.690*** (0.0893)
logcn	β_2	0.456*** (0.0144)	0.472*** (0.0182)
year	β_3	-0.00555* (0.00232)	-0.00886** (0.00303)
_cons	α_i	17.02*** (4.780)	23.49*** (6.075)
mu	μ	1.515*** (0.244)	1.499*** (0.233)
sigma_u	σ_u^2	0.4369	0.4314
sigma_v	σ_v^2	0.0574	0.0572
sigma ²	$\sigma_s^2 = (\sigma_v^2 + \sigma_u^2)$	0.4943	0.4886
gamma	$\gamma = (\sigma_u^2 / \sigma_s^2)$	0.8839	0.8829
lambda	$\lambda = (\sigma_u / \sigma_v)$	2.7589	2.7460
eta	η	-	0.0015
log likelihood	-	-103.0317	-101.9282
observations	N	1472	1472

Source: Own elaboration.

With the discussion of the variabilities, the results return positive values of σ_u^2 / σ_s^2 , which are ca. 48% and ca. 49% for time-invariant and time-varying decaying inefficiency models, respectively. These values suggest that within the time frame under consideration, technical inefficiencies accounted for the differences between the actual output (real GDP at chained PPPs) and the production frontier (potential output and not random shocks alone). On a yearly basis, this translated to an average efficiency score of ca. 40% and 26%, respectively, meaning that SSA countries had 60% and 74% chance to reach their maximum output potential. The maximum likelihood results also return ratios of gamma (γ), which are ca. 88% and ca. 88%, respectively. The interpretation of these ratios is that, 88% of random variability of the outputs of these countries, is due to technical inefficiency when analyzing the data under the truncated normal distribution. Furthermore, we initially made the assumptions of differences in economic structures and policies. The lambda values of 2.7589 and 2.7460, respectively, indicate differences in actual production or output due to differences in economic structures, resources, and

other factors such as economic policies and managerial abilities rather than random variability. The estimator of the parameter of time-varying decay η of ca. 0 indicates that the model reduces to the time-invariant model, making it not warranted when considering this data in applying stochastic production frontier analysis under the truncated normal distribution to analyze the efficiency of the selected SSA countries.

Efficiency scores were also estimated to compare the competitiveness of the selected SSA countries. Table 3 presents their mean technical efficiencies under the time-invariant model. Appendix A and B also contain information on the technical efficiency of the selected SSA countries.

Table 3. Efficiency results for selected SSA countries in 1980–2019

Rank	Country	Technical efficiency	Rank	Country	Technical efficiency
1	Equatorial Guinea	0.93	23	Mauritania	0.36
2	Mauritius	0.77	24	Congo	0.36
3	South Africa	0.73	25	Ghana	0.36
4	Eswatini	0.65	26	Uganda	0.36
5	Gabon	0.65	27	Zambia	0.36
6	Sudan	0.61	28	Lesotho	0.36
7	Botswana	0.61	29	Cabo Verde	0.33
8	Namibia	0.55	30	Burkina Faso	0.31
9	Seychelles	0.55	31	Benin	0.31
10	Zimbabwe	0.55	32	Chad	0.29
11	Guinea	0.49	33	Mozambique	0.29
12	Djibouti	0.46	34	Madagascar	0.28
13	Côte d'Ivoire	0.45	35	Malawi	0.27
14	Gambia	0.41	36	Togo	0.25
15	Mali	0.41	37	Nigeria	0.25
16	Cameroon	0.40	38	Guinea-Bissau	0.23
17	Angola	0.38	39	Ethiopia	0.21
18	Senegal	0.38	40	D.R. of the Congo	0.20
19	Sao Tome and Principe	0.38	41	Liberia	0.20
20	Rwanda	0.37	42	Burundi	0.20
21	Kenya	0.37	43	Central African	0.18
22	Sierra Leone	0.37	44	Niger	0.18

Source: Own elaboration.

The efficiency scores indicate the average potential output of these countries. The average realized potential output for all 44 SSA countries was 0.40 with a stan-

standard deviation of 0.17 for the time-invariant model. This score indicates that the selected countries can improve their efficiency levels by 60%, all things being equal. Equatorial Guinea, Mauritius, South Africa, Eswatini, and Gabon were found to be highly competitive, while Congo, Liberia, Burundi, Central Africa, and Niger were at the bottom of the ranking.

Conclusions

This study utilized the estimation method of the stochastic frontier model through the framework of Cobb-Douglas production function to evaluate the competitiveness of 44 selected SSA countries. The results show that Equatorial Guinea, Mauritius, South Africa, Eswatini, and Gabon are highly competitive. In contrast, Congo, Liberia, Burundi, Central Africa, and Niger were found to be less competitive based on their efficiency scores when utilizing the data under the truncated normal distribution.

On average, SSA countries realized potential outputs of 40% based on model 2 under truncated normal distribution. The interpretation of these efficiency scores is that, on average, SSA countries have the potential to improve their efficiency levels by 60% and thus increase their competitiveness.

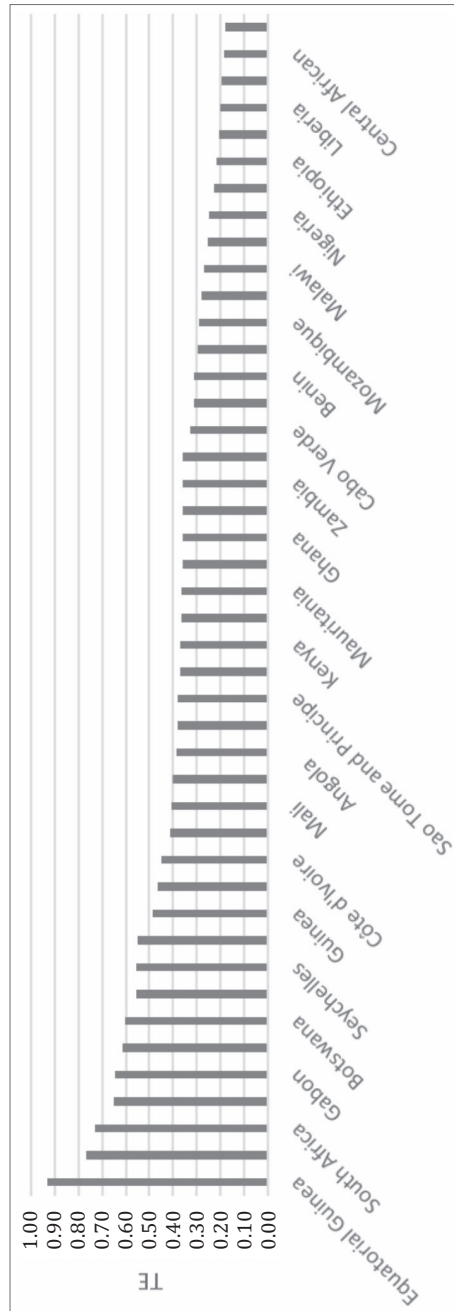
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Appendix A. Average technical efficiency for selected SSA countries in 1980–2019



Source: Own elaboration.

Appendix B. Average yearly technical efficiency for SSA countries in 1980–2019



Source: Own elaboration.