PROBLEMY WCZESNEJ EDUKACJI / ISSUES IN EARLY EDUCATION ISSN 1734-1582
2 (57) / 2023 ISSN 2451-2230 (online)

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https://doi.org/10.26881/pwe.2023.57.09

What is the area of a pig? Problem posing and problem solving in early childhood education

Summary

Problem posing and problem solving serve as a crucial element of classroom instruction in early mathematics education, and have long been a topic of study of many practitioners and researchers. Used as a powerful tool for differentiation, they affect the ways in which the practice of mathematics is perceived by students and also help teachers gauge children's understanding of concepts. In the student-centered approach, problem posing and problem solving can successfully engage students in creative educational situations. This research has been part of a broader design research project, the aim of which was to investigate the relationship between a classroom environment that allows for dialogue and young children's propensity to design and solve their own tasks. Methodology included taking field notes and photographs, followed by reflection sessions. Turning young mathematicians into independent inquirers helped them gain authentic ownership of their knowledge. Additionally, it aided in the development of the young children's competencies in effective engagement in problem posing activities. The toolbox of instructional techniques for problem posing in the classroom evolved, transforming mathematical classrooms into inquiry polygons for all learners.

Keywords: dialogic teaching, early childhood mathematics education, problem posing, problem solving

Słowa kluczowe: uczenie dialogujące, matematyczna edukacja wczesnoszkolna, formułowanie problemów, rozwiązywanie problemów

The mind uses its faculty for creativity only when experience forces it to do so.

Henri Poincaré

Introduction

The importance of solving problems in elementary school mathematics classrooms derives from the fact that most mathematical concepts can be taught through carefully designed problems (NCTM 2000: 52). Teachers who understand this simple truth can use and adapt tasks that will lead students towards anticipated learning goals. Fostering mathematical inquisitiveness through problem solving can develop human curiosity and, hence, lead to the formulation of new problems to be solved. The idea of asking questions and posing problems is crucial at every stage of mathematical education. “Problem posing refers to both the generation of new problems and the reformulation of given problems. Thus, posing can occur before, during, or after the solution of a problem” (Silver 1994: 19). The reformulation of a problem refers to a heuristic strategy described by Pólya (Pólya, Conway 2014: xvi–xvii), and entails turning a complex problem into a simpler variation that is more accessible to the solver.

Designing a task is an excellent way to reveal the level of concept understanding and language development; it is also a measure of creative abilities. It can serve as a means to provide a challenge and promote mathematical sense making. Teachers are able to foster young learners’ mathematical disposition by selecting relevant problems and by offering space for utilizing the students’ natural gift for posing problems.

Problem posing should be considered from the teacher’s perspective, as well as from the perspective of a student. In the case of the former, problem posing can be used as an instructional task or problems can be posed for the students to solve. From the perspective of the student, they may be asked to pose problems with predefined conditions or to redesign existing problems (Cai, Hwang 2020). This article will tackle different classroom situations which present both the teacher’s and the students’ perspectives.

Creating a problem-solving atmosphere requires providing problem-centred activities that offer opportunities for children to participate in “interactive situations in which sharing their ways of thinking is encouraged and respected” (Wood 1993: 12). These activities allow for students to construct concepts and operations of progressively advanced character; they promote the development of independence in approaching challenging and open-ended problems.

Students’ powers of communication can be improved by implementing dialogic teaching, which is guided by six principles offered by Robin Alexander (2020: 131):

- Collectivity – joint learning and enquiry performed in groups or as a class;
- Supportiveness – feeling able to freely express ideas, helping each other to gain common understanding;
- Reciprocity – listening to each other, sharing ideas, considering alternative viewpoints;
- Deliberation – presenting and evaluating arguments, working towards reasoned positions and outcomes;
- Cumulation – building on their own and each other’s contributions, chaining them into coherent lines of thinking and understanding;
- Purposefulness – talk is designed to gain specific educational goals in view.

Dialogic teaching emphasizes the value of responsible learning – negotiating and co-constructing meanings through dialogue. It invites students to become thinkers and reasoners who actively participate in creating the culture of argumentation. “Becoming a good mathematical problem solver – becoming a good thinker in any domain – may be as much a matter of acquiring the habits and dispositions of interpretation and sense-making as of acquiring any particular set of skills, strategies, or knowledge. If this is so, we may do well to conceive of mathematics education less as an instructional process (…), than as a socialization process" (Resnick 1988: 58). By creating a dialogic space, we facilitate the gaining of new argumentation skills, so students may strengthen their intellectual abilities and become more successful learners.

Methodological framework

Research problem and research questions

Within the framework of the dialogic teaching approach, collective learning and inquiry are promoted not only as social and emotional acts, but also as the ultimate achievement in the process of purposeful classroom talk. Hence, the main research problem was stated as follows: What is the relationship between a classroom environment that allows for dialogue and young children's propensity to design and/or solve mathematical tasks?

The variable of interest in this inquiry was the quality of responses students gave to problem posing over the course of two semesters. Moreover, the author aimed to answer the following questions:

- What are the best instructional strategies to help promote productive problem posing?
- What are effective strategies for scaffolding language?
- How can the idea that understanding the solving process is the essential part of problem solving be reinforced?

Research structure

Throughout the research process, a student-centered and dialogic teaching approach was implemented. The teacher-researcher worked with six groups of elementary school children in grades 1 to 3, teaching 30 hours of mathematics per week during the course of one school year in a private school in Warsaw, Poland. A concise and comprehensive definition of **design research** is given by Bell, who described it as "those enterprises that involve intentional design coupled to empirical research and theorizing about what takes place in the authentic contexts where the designed objects come to be used" (Bell 2004: 245). According to Freudenthal, this research method is "experiencing the cyclical process of development and investigation and describing this experience in an honest, self-justifying way, so that this experience passed on to others can become their own" (Freudenthal 1991: 161). The importance of this kind of research is the combination of theory and practice. The researcher-theorist plans actions to bring about a solution to a problem, then tests the ideas supported by the theory in the environment affected by the problem. The main question she tries to answer is: “Will it work?” rather than “Is it valid or true?” Design research seeks to construct knowledge that is both actionable and open to validation.

The aim of design research is to develop the means, tools, tasks, materials and activities that will serve the learning process and bring a remedy to the problematic situation at the center of the research action. The collection of the developed methods is the final product of the researchers’ efforts (Mintrop 2016: 16).

In the course of the research process, the researcher took field notes and gathered photographs of examples of students’ work. Cycles of reflection followed lesson series and served as cognitive tools to navigate the process of designing and redesigning activities. The number of classes and the number of pupils, together with a breakdown by gender, are shown in the Table 1.

Table 1. Number of girls and boys in the classes in which the design research was conducted

School Year 2022/2023		
Class grade	**Girls**	**Boys**
Two Year 1 classes	17	10
Two Year 2 classes	11	12
Two Year 3 classes	15	11

Source: Author’s research.

This design research is described by a **hypothetical learning trajectory** containing learning objectives, a description of the activities and the anticipated outcomes of their implementation, as well as their relevance to the students’ reasoning and thinking processes. The hypothetical learning trajectory relates to teaching as an exploration of various aspects of the teacher’s knowledge, setting objectives and direction of the activity to be followed by the trajectory of the educational process and the students’ mathematical learning and thinking (Simon 1995). Initially, the process of constructing a hypothetical teaching/learning trajectory was associated with the design of single lessons, but was later extended to include series of activities of a similar nature. Proposals for corrective changes were implemented, reflected upon, revised and expanded with new elements (Table 2).

Table 2. Hypothetical learning trajectory defined in the present design research

Features of collaborative classroom culture	Designed activities for the students	Students’ dimension	
		expected outcomes	real outcomes
Interactive settings	Problem posing in groups	Collaborative metacognition developed	Asking metacognitive questions about one’s thinking to others

Features of collaborative classroom culture	Designed activities for the students	Students' dimension	
		expected outcomes	**real outcomes**
Cooperative goal structures	Presenting individual/ group work results	Evolution of presentation skills, number sense development	Confidence in presenting ideas, handling questions number sense developed
Emphasizing the solving process	Explaining the solving process	Discussing steps of a solving process	Reflection on strategies
Using strategies for scaffolding language	Whole class discussions	Mathematical language improvement	Paying attention to clarity and precision of mathematical language
Metacognitive interventions	Answering metacognitive questions	Metacognitive awareness enhancement	Providing justifications and reasons

Source: Author's research.

The results of this research will be presented in the form of the analysis of examples of problems that were posed and/or solved by the students with reference to real outcomes in the students' dimension.

Results

Problem posing in groups

Problem 1. Pose a problem with fractions (Year 2)

Both examples (Fig. 1, 2) that are presented below come from of a set of group work results. Each group was asked to discuss a posed problem at the board.

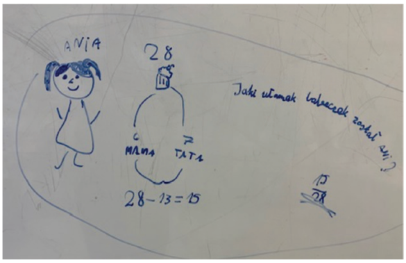

Figure 1. Year 2 work example (group of girls)

Source: Author's archives.

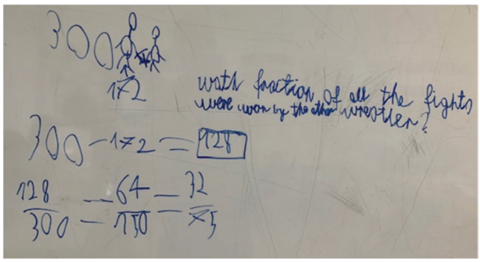

Figure 2. Year 2 work example (group of boys)

Source: Author's archives.

Figure 1 illustrates the following problem created by a group of girls:

> Ania baked 28 cupcakes. She gave 6 of them to her mom and 7 of them to her dad. What fraction of all the cupcakes was left for Ania?

Figure 2 illustrates the following problem posed by a group of boys:

> Two wrestlers had 300 fights between them. One of them won 172 fights. What fraction of all the fights was won by the other wrestler?

In both cases, the students exhibited a full understanding of the concept of fractions. Solving the first task required an extra step, which made it more engaging. The second group decided to simplify the final fraction, which was quite a challenge.

Problem 2. Pose a problem to a given equation (Year 2): 639 − ? = 96

One example of a problem that was posed by a group of students was as follows:

> A florist had 639 flowers. He lost some of them and the next day had only 96. How many flowers did he lose?

The extension of the task was to pose additional problems without a prompting equation but with the use of the students' own equation (Fig. 3). Here we have two examples of group work results:

> There were 149 apples on a tree. A monster ate 85. How many apples are left?
>
> There were 583 pencils in a classroom. Some of them went missing and now there are only 240 pencils left. How many pencils went missing?

Different cognitive processes underlie the two types of problem-posing tasks above: in the first case, the process of comprehending and organizing quantitative information was involved, in the second case, it was the process of editing quantitative information. All of the problems posed could be solved by using three different strategies of the students' choice: adding up on the number line, subtracting by splitting a subtrahend to tens and units, and splitting both minuend and subtrahend. When children were asked for the justification of their solving strategy choice, they answered that they "wanted to find the easiest way to bring the result." The first group expressed the likeness of a number line and the need "to see how the solution is created." With these three examples, the other students in the class agreed that the chosen strategies worked well for the given tasks. Answering metacognitive questions such as "Why did you do it this way?" or "Why did you choose this strategy?" helps the students realize that they know different ways to tackle the task and that they have a choice (and a voice) in their mathematical endeavours.

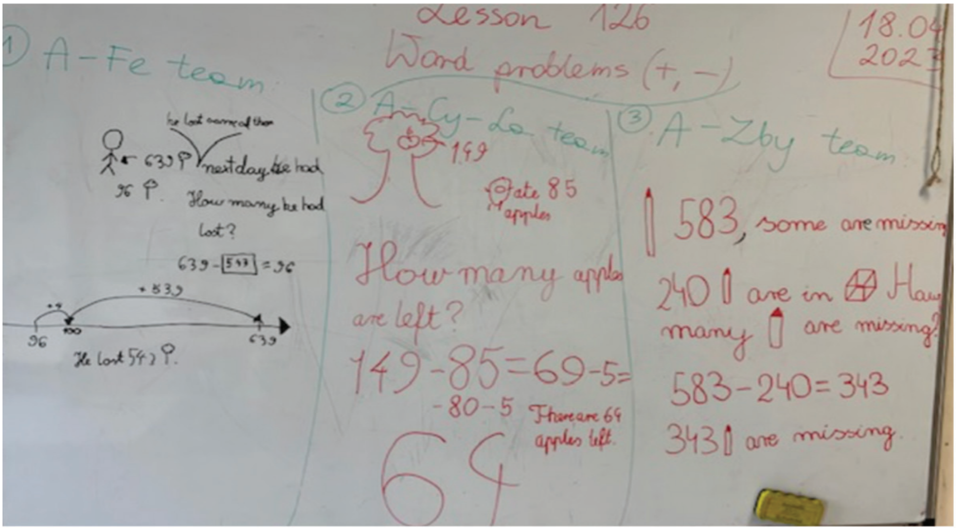

Figure 3. Children’s tasks with solution (Year 2)

Source: Author’s archives.

Presenting individual/ group work results

For the purpose of the further analysis, the term “number sense” must be defined. According to Howden, “number sense can be described as good intuition about numbers and their relationships. It develops gradually as a result of exploring numbers, visualizing them in a variety of contexts, and relating them in ways that are not limited by traditional algorithms” (Howden 1989: 11). Providing an environment that fosters curiosity and exploration is essential for the development of number sense. An important component of the students ‘problem-solving process is formulating follow-up questions that help in organizing their thoughts and in ensuring that they fully understand the posed problem. The two following problems posed by the teacher successfully prompted the students to ask follow-up questions and, therefore, to reach a deeper understanding of the problem.

The first problem presented to the students had to do with Number Jumbler, which is a mathematical game consisting of seven dice locked in a plastic container called a jumbler. The object of the game is to get the sum of the numbers on the five coloured dice as close as possible to the sum of the numbers on the two black dice. The numbers on the dice are changed by rolling the **Number Jumbler** against the palm of one’s hand. Each coloured die’s number can be used only once as part of any possible mathematical operation in any combination or order.

Problem 1. Try to reach 36 (the target number) by using 1, 1, 5, 4, 1 and any operation (addition, subtraction, multiplication, division) (Year 1)

Before working together in groups, the students had a chance to ask questions about the problem at hand (Fig. 4). The following are a few examples of their questions:

> Do we need to use all the numbers?
>
> If we add 1 and 1, can we then add 2 to 5 and multiply 7 by 4?
>
> Can we stay with one calculation only?
>
> What happens if we multiply by 1?
>
> Does it make a big difference if we add 1 + 1 + 1 and multiply it by 5 + 4?

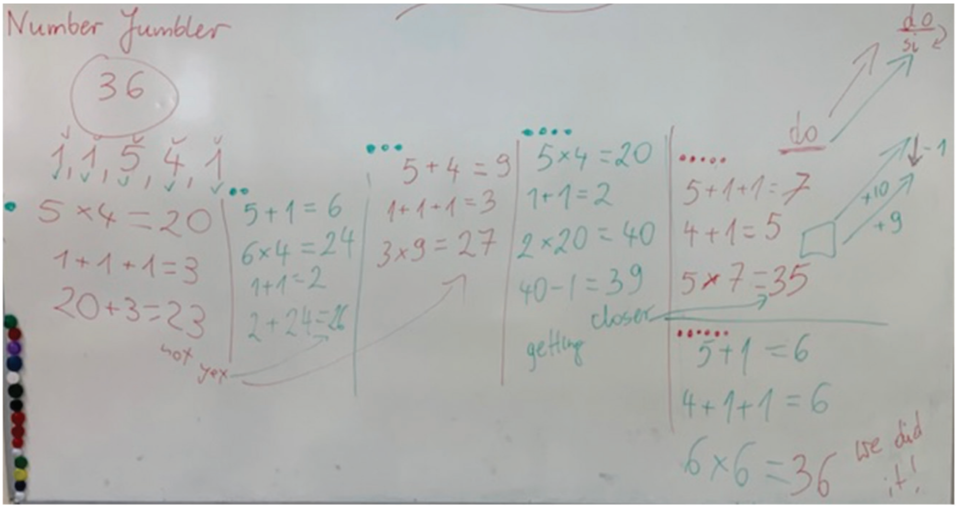

Figure 4. Students' solutions for reaching a target number in the Number Jumbler game

Source: Author's archives.

Every time a new group was presenting their findings, the teacher was trying to build a positive tension in the classroom by making comments like: "not yet" or "getting close" and finally "We did it!" The whole class was very much engaged in the activity and many children wanted to take a new target number to try to tackle the problem on his/ her own at home. Here is one of the examples of solutions to the new target number of 42 and 2, 5, 2, 3, 3 as numbers to be used to reach it. The first grader was able of going through a systematically designed process leading to the solution of a complicated task. First, the girl used 2 × 5 = 10, 10 × 2 = 20 and 3 × 3 = 9. This is when she realized that the target number couldn't be reached that way. The second attempt was 3 + 3 = 6, 6 + 2 = 8, 8 × 5 = 40 and 40 + 2 = 42, which brought her to the goal.

Nevertheless, the girl continued her search and found another way to reach the target number, which was: $5 \times 2 = 10$, $10 - 3 = 7$, $3 \times 2 = 6$ and $7 \times 6 = 42$.

Such actions, initiated by the students themselves, create an inspirational experience for the rest of the group and serve as a means of improving students' number sense. The autonomy of learning is boosted as a result of such situations.

Problem 2. The perimeter of a rectangle is 20 cm. What could its length and width be? (Year 2)

The second graders were already familiar with 2-D shapes, yet this type of an open-ended task posed by the teacher was a great challenge. Group collaboration in testing this geometrical situation was very lively and the groups were eager to share their findings.

Not all of the groups knew right away how to tackle the problem. Some of the children started adding four numbers to reach 20 as total. Then, they realized that this is not right because a rectangle has two pairs of identical sides. One of the students realized that 20 cm consists of 2 parts of 10 cm each. After that the groups started working with number bonds to 10 and the results were as follows: $P = 2 \times 6$ cm $+ 2 \times 4$ cm $= 20$ cm, $P = 2 \times 7$ cm $+ 2 \times 3$ cm $= 20$ cm, $P = 2 \times 8$ cm $+ 2 \times 2$ cm $= 20$ cm, $P = 2 \times 9$ cm $+ 2 \times 1$ cm $= 20$ cm. For a moment the children were confused with two following results: $P = 2 \times 10$ cm $+ 2 \times 0$ cm $= 20$ cm and $P = 4 \times 5$ cm $= 20$ cm. The first one didn't make sense because "this wouldn't be a rectangle!" The second equation brought doubts because as someone stated: "But this is a square, not a rectangle!" Only answering the teacher's question: "Does a square have two pairs of identical sides?" helped the students to realize that a square is also a rectangle. The whole process: posing a question, group work, results presentations and whole class discussion lasted for two lessons. At the end, the students were asked to reflect on the steps that were taken. This is how they described the route leading through the process:

1. First, we talked about what a rectangle is and how we calculate its perimeter.
2. [In groups] we tried to find numbers that give 20.
3. X said that we have to look for numbers that give 10 because these two parts are the same [the child meant the total of the length and the width].
4. We found number bonds to 10 and added them up.
5. We had a talk about 0 cm by 10 cm rectangle and we could see that it wasn't a rectangle.
6. We explained why a square is a rectangle.

These statements were offered by the individual group members who volunteered to contribute to the discussion. Through the reflection on the solving process, everybody has a chance to make sure that he/she has the full understanding of what was done by the group and by the class. The teacher may ask specific group members to contribute to the final discussion. All the children know that at the end of group work time, everybody should be ready to talk. The groups are usually chosen by the teacher with mixed ability feature as a characteristic. If the students insist on their individual choice of groups and one of the teams finishes solving the original problem, they may be asked to extend the task or to generate an alternative one.

Open-ended tasks give many opportunities for verification of students' concept understanding and at the same time, they bring the scaffolding process to the teacher's attention.

Explaining the solving process

After introducing the concept of area by using different sized unit squares (1 mm^2, 1 cm^2, 1 dm^2 and 1 m^2) and measuring the area of various objects, the students were supposed to solve a task posed by the teacher.

Problem 1. Design your own irregular shape and estimate its area (Year 3)

Figures 5 and 6 show two examples of the results of individual work in response to the posed problem and can demonstrate the mature thinking abilities of the children.

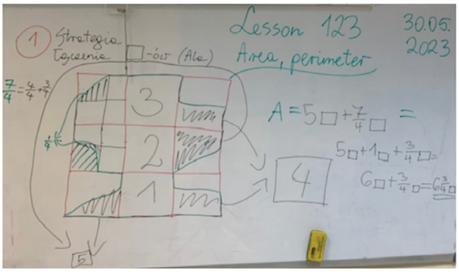

Figure 5. Year 3 students' work samples

Source: Author's archives.

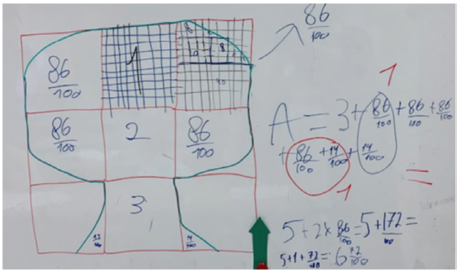

Figure 6. Year 3 students' work samples

Source: Author's archives.

In the first case the strategy was called "joining the squares." It was described as "first looking for the whole squares and later trying to combine parts of the squares to form either full squares or the biggest possible part of a square." The units in which the area was expressed were simply called squares. The first 3 squares could be obtained in the obvious way by counting full squares, square number 4 contained two halves, square number 5 was a combination of the two smaller shapes that complemented each other to form another full square. Finally, for precision purposes, the squares were divided into quarters, four quarters turned into one square, then the other two smaller parts estimated to ¼ of a square and with the two other quarters added to ¾, so finally the whole area becomes 6 and ¾ of a square.

The second example shows a shape of a tree. The student wanted to work on the estimation in the most precise manner. Because of that, after counting up the first 3 full squares, the other squares were divided into smaller units (100 smaller units becoming 1 big unit as in 1 cm^2 = 10 mm × 10 mm = 100 mm^2). By counting the missing parts of the square as 14/100, the other part could be counted as 86/100. Four parts of that size are present, the other two by the tree trunk have identical sizes of 14/100. That way the final calculations are P = 3 + 86/100 + 86/100 + 86/100 + 86/100 + 14/100 + 14/100 = 5 + 172/100 = 6 and 72/100. The square symbol is missing in the calculations but the students kept referring to a square as a unit.

In both cases the students exhibited a very deep understanding of the concept of area, they were also able to explain their ideas very clearly, so the rest of the class was fully satisfied with the explanations about the problem-solving process.

Problem 2. Estimate the area of an animal silhouette (Year 3)

Problem situation was created by providing the children with woodblocks with animal shapes. After choosing one of the blocks, the students were supposed to outline the shape on a graph paper and calculate its area. The graph paper had either 1 cm^2 or 1 mm^2 size squares. One of the tasks generated by the students was:

> What is the area of a pig? (Year 3)

The students chose 1-mm^2 grid paper because as they stated: “that would give them the more precise answer.” They analyzed the pig print by dividing the whole shape into squares of 1 cm^2 and then, within these squares, they dealt with smaller fragments by using a smaller unit – squares of 1 mm^2. After completing their count of full 1 cm^2 squares (8 m^2), they added the parts of the squares by adding the tiny squares expressed as 1 mm^2 units: 85/100 + 85/100 + 86/100 + 75/100 + 73/100 + 18/100 + 12/100 = 4 and 34/100 (the last row of calculations), 87/100 + 14/100 + 13/100 + 95/100 + 21/100 = 2 and 20/100 (that should be 2 and 30/100), 21/100 + 79/100 + 70/100 + 86/100 = 2 and 56/100 with all three results added up to 17.10 cm^2. Although a small mistake was made during the addition process (the perimeter should be 17.20 cm^2), one can see that the solving process was carried with mathematical clarity and elegance (Fig. 7).

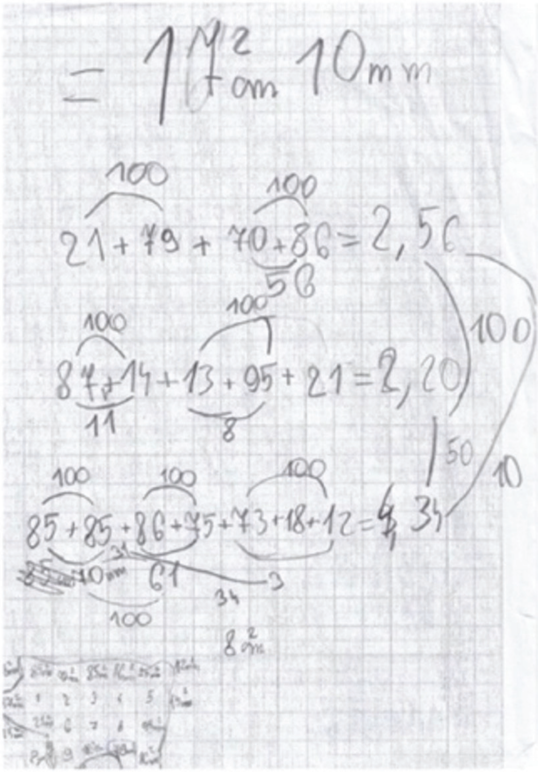

Figure 7. The solution of the problem presented by Year 3 students.

Source: Author's archives.

Whole class discussions

Problem 1. Does 54 − 27 = 57 − 24? (Year 1)

Children in Year 1 were asked if 54 − 27 gives the same result as 57 − 24. They needed to justify their decision. Originally many students answered "yes" to this question, but some others didn't agree. Then the discussion started. Somebody decided to split the numbers into tens and units and then perform subtractions accordingly: 50 − 20, 4 − 7 and 50 − 20, 7 − 4. Because 50 − 20 = 30 in both cases, the children realized that the final question was: "Is 4 − 7 the same as 7 − 4?" Someone said that 4 − 7 = 0 and 7 − 4 = 3. Not everybody agreed with the first statement. Then one of the children came to the board and using a number line demonstrated that 4 − 7 = −3. After being asked if both results (0 and −3) can be correct, the students decided that only one answer: either 0 or −3 can be true, because there is only one truth. At this moment one girl explained: "Yes, there is only one truth, but the way we perceive it depends on how much we know, because if somebody has no idea that we have negative numbers, then this person cannot decide that −3 is true.” The students agreed with that and were very content that they had a chance to discover numbers “on the left side of a 0.” This example can give us a genuine insight into the mind of a child. The children love to run investigations and they are very eager to search for truth. Their philosophical disposition can and needs to be challenged during regular mathematics lessons.

Replacing the original problem “Is 54 − 27 = 57 − 24?” by “Is 4 − 7 = 7 − 4?” was an example of applying reformulation (problem posing). As a result, the children found a simpler form of a task to work on. This is one of the heuristic strategies for problem solving.

Problem 2. What number could be used in the box (i.e., in the denominator) so the given number sentence makes sense? 1/□ of 100 =

Try to look for as many solutions as possible. (Year 2)

After solving the open-ended task posed by the teacher, the students were supposed to work individually or in groups to generate a problem by varying a single element (number 100) of the original. There was an additional condition given to the children in this investigation – they were supposed to try to notice something interesting about their solutions (Fig. 8).

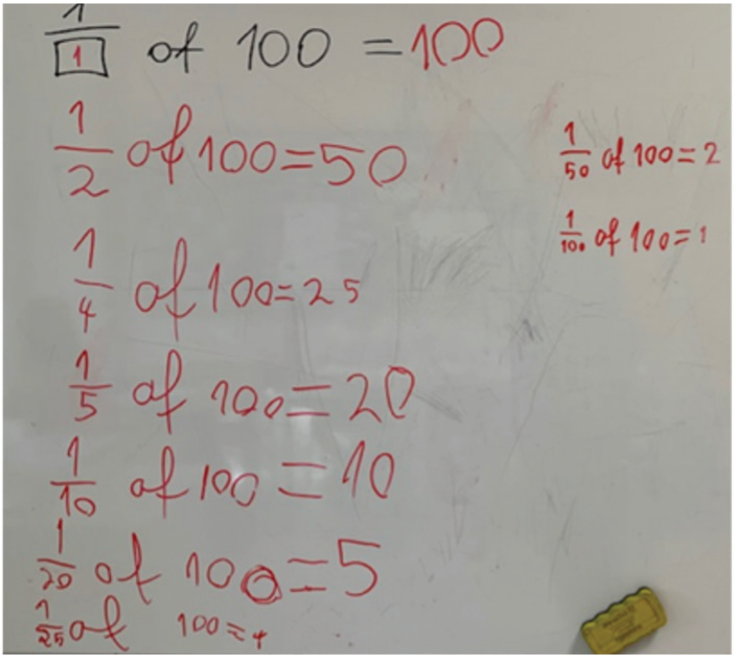

Figure 8. Set of solutions to the original problem (Year 2)

Source: Author’s archives.

Year 2 students had a chance to explore the solutions to the task and then they needed to choose their own number to continue the exploration. They tried to find the number which could give them the greatest number of solutions (Fig. 9, 10).

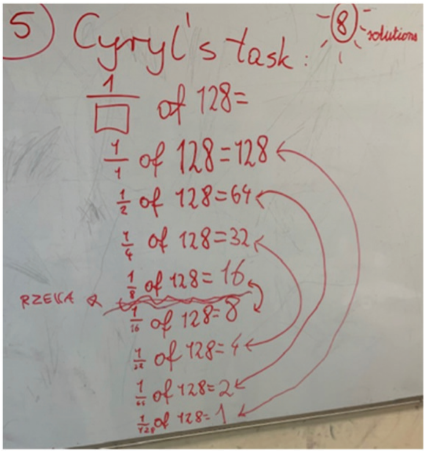

Figure 9. 1/□ of 128 =

Source: Author's archives.

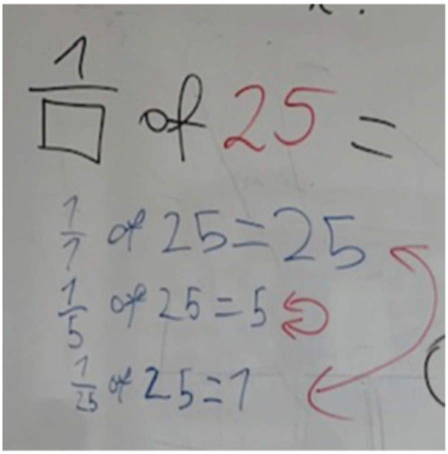

Figure 10. 1/□ of 25 =

Source: Author's archives.

The children realized that the missing denominators ("numbers at the bottom") have to be factors of the number of their choice. Then they started discussing a pattern among the set of denominators and the set of solutions and they figured out that the created number sentences could be paired. The first time that it happened, the students didn't pay attention to the order of solutions, but later they were trying to work systematically to design symmetrical patterns. Someone decided that in the middle there is a "river" (RZEKA) that separates the two parts. At this point somebody discovered that for some numbers, like 25, the number of factors wasn't an even number, so the class became excited about looking for some other numbers of that type and thanks to that, the whole class could discover square numbers.

Solving this type of mathematically rich open-ended task may give the children a lot of satisfaction. It leads to discoveries of the new layers and meanings. Language scaffolding was a very important part in the solving process. Many times, the students needed to describe the concepts they were discussing and they were happy to be offered the forgotten names such as a factor, denominator, or symmetry. Using proper vocabulary made their life as explainers much simpler.

Problem 3. How do you find the perimeter of a curved shape? (Year 2)

The second graders were already familiar with the concept of the perimeter of polygons. They were supposed to work in pairs, and each group was given a paper with a shape. Children could also pick any object from a given set to solve a problem: a ruler, a measuring tape, a marker, a ring, a bottle top, a string. After the group work, the students had a chance to present their ideas. A whole set of solving strategies was created:

1. Use a measuring tape (place the measuring tape along the shape and measure).
2. Use fingers – spread them at a distance of 2 cm, count how many 2 cm parts we have (divide the shape into small sections of the same length, multiply the number of pieces by the length of a piece – the smaller they are, the more precise the result).
3. Step-by-step with a ruler, try to use every new direction (divide the shape into smaller pieces trying to measure parts that are close to segments, then add the lengths of the parts).
4. Use a bottle top and travel around the shape (count the number of full turns and find the circumference of the bottle top, multiply).
5. Place a string on the shape then use a ruler to measure the string.
6. Use a string to make a measuring tape by marking centimeters and count.
7. Use a string to measure parts of the shape, measure what is left (measure parts of the shape with a string, add them up).

Offering a clear and precise description of the solving strategy was a challenging task for the groups. It required a lot of scaffolding of the language to formulate the final "recipes" for different strategies. The text written in italics (above) is the original form of the students' instructions. The final versions (after negotiating the meanings and clarity with the whole class) are written in parentheses. Some examples of Year 2 group work are presented below, in Figures 11–13.

Figure 11. Strategy number 2

Source: Author's archives.

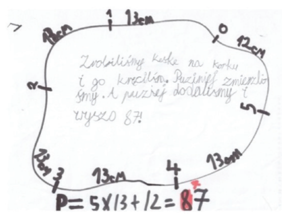

Figure 12. Strategy number 4

Source: Author's archives.

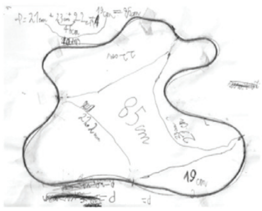

Figure 13. Strategy number 7

Source: Author's archives.

Answering metacognitive questions

Discussing metacognitive processes may be easily exposed when analyzing mistakes. This invaluable learning tool to be used with the children is not to be emphasized enough.

Problem 1. Compare these three number sentences: 38 + 18 = 20, 38 − 18 = 20 and 38 + 18 = 20. What can you see? (Year 1)

The children knew that something wasn't right. They were working in groups and afterwards shared their ideas with the whole class. Somebody explained that only the second number sentence had the right result, but then the children were supposed to discuss how it happened that the mistakes in the other two cases occurred. That was interesting to watch children's engagement.

> Student: In the first situation somebody made a mistake and instead of minus used plus.
>
> Teacher: How did this mistake occur?
>
> S: That person didn't pay attention.
>
> S: In the last case someone wanted to split the numbers and got 3 + 1 = 4 and 8 + 8 = 16 and added these two results.
>
> T: How did this happen?
>
> S: First the person didn't know that 3 in 38 and 1 in 18 were tens and then he added 4 to 16 which was good, but the final result couldn't be right because of this "mixed up" mistake.
>
> T: Was it the same type of mistake that in the first example?
>
> S: No, here that person didn't understand.
>
> T: Okay, so that was a good mistake, because thanks to our discussion we made sure that everybody was on the same track. That was a GOOD MISTAKE.

Recognizing good mistakes by Year 1 students can be a real treat for the teacher. After this incident, some of the children wanted to become Good Mistake Detectives, a couple of weeks later the majority of the children in both classes were ready to join them in their search.

Students can be very reflective about their own mistakes. That gives the chance to the teacher to make sure that the scaffolding process takes place in the classroom. Some examples of classroom interactions can be seen in the table 3.

Table 3. Examples of teacher-student exchanges related to mistakes

Student's statement	Teacher's response
"I always make the same mistake."	"So now you know when to be more attentive."
"Why did I make this mistake?".	"Take a few steps back in the solving process and maybe you will find the answer to your question."
"It is always easier to find other people's mistakes."	"That's why I like you to check your work together with a partner."
"Correcting my own mistakes makes me become more attentive later on."	"That's why it is a good habit to have the mistake marked but not corrected by the teacher."
"Correcting other people's mistakes makes me more aware of why the mistakes were made."	"Exactly, you start analyzing the mistakes and you make sure that you understand the concepts."

Source: Author's archives.

Analyzing some examples of "good mistakes" helps the children to start paying attention to the value of mistakes and to the importance of the solving process. Another strategy could be to ask the students to check their work in pairs or in groups. They are usually very eager to look for other people's mistakes. One can also give the children samples of work with mistakes and to analyze them together in a class discussion or to ask the children to prepare tasks with mistakes but to have the explanations for making these mistakes. By guiding the students, one can help them to enhance their metacognitive awareness. At the same time, it may become clearer to the students why they should pay attention to the steps of the solving process.

Conclusions

In the dialogic classroom, activities are organized around tasks fostering curiosity and exploration. Group work promotes collaborative learning which impacts students' agency. Teacher's responsibility is to provide challenging tasks, support effort and encourage children's contribution. Participating in dialogue enhances students' potential, helps them discover new concepts and enables to exchange meanings. Teacher's role is also to provide framework to bring children's understanding and knowledge development to the next level.

Students in early childhood education are full of creative energy. Their open minds are bursting with ideas which can be utilized during exploratory talk. Their imagination can vent through asking questions and posing problems that can be initiated through many instructional strategies. The first encounters with numbers can conveniently serve as problem situations. Children may be asked to think of a story in which the given number is present. With the evolution of arithmetical skills, one can ask the students to think of a story that will be solved with a specific arithmetic operation. Usually when posing problems, students may be thinking about solutions, so giving them an equation, a graph or a model (a number line) to build their problem around it, can be quite helpful. After solving a problem posed by a teacher, children may be asked to extend a problem, to generate a task by varying a single element or to design a problem related to the original one. The teacher may also try to ask questions like: “What else can we ask about now?” or “Can you think of changing the situation in the original story so a different question may be asked?”

With the experience gathered by the children, the complexity of problems and the clarity of language arise. Scaffolding language development requires special attention from the teacher and it can be obtained by:

- encouraging students to create mathematical statements independently;
- rephrasing students’ statements to point out the proper language use;
- asking other students to rephrase someone’s statement;
- reviewing vocabulary (important phrases) before group work/discussions;
- referring to the vocabulary that is presented in the classroom (the word you are trying to describe is on the board);
- asking students to improve their language (How can you rephrase your statement?).

That way the independence is not taken away from the students but the conditions for growth are provided. Scaffolding norms of inquiry classroom can also transform the quality of the collaborative learning environment. Negotiating concept understanding and making sense of the solving process can be introduced by asking the students questions:

- What did you do?
- Can you tell us more about it?
- Do we all agree with what X just said?
- How can we explain this solving strategy?
- Who can take over the explanations from that point?

The teacher can use such questions to initiate a meaningful talk not only during the whole class discussion but also while moderating group work. Reflecting on the solving process after the activities is crucial for students’ understanding that not only the final result is in focus. Sharing incomplete ideas and “reasoning that operates on the reasoning of another” (Berkowitz, Gibbs 1983: 402) can become everyday classroom experience. It can help the children realize that mathematical inquiry is about collaboration which brings everybody to a better understanding. The reflection can occur by:

- Going through the steps of the solving process (What were the steps? How was it done?);
- Discussing the difficult parts of the solving process (Which part of the solving process was difficult to you and why?);
- Reasoning about the choice of the most effective strategy (Why did you choose this strategy for this problem?);
- Analyzing mistakes /looking for good mistakes.

Experiencing the joys and struggles of problem posing and problem solving can give the students a sense of immersion in authentic mathematical reality. We provide the students with more human version of mathematics and offer them space to add their personal touch to it.

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