THE EFFECT OF FINANCIAL RISK ON THE DEMAND FOR SUPPLEMENTARY HEALTH INSURANCE

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Abstract

As access to medical services provided by public healthcare system in Poland is limited, private supplementary health insurance becomes an increasingly interesting option for patients. Private insurance market has grown significantly in recent years. Therefore, it is interesting to determine factors contributing to the demand for supplementary health insurance by adopting a mathematical model of both compulsory public health insurance and additional, voluntary private health insurance. Such a model was introduced in Dudzinski’s work (2015). The model takes into consideration such factors like compulsory insurance premium, health status of a patient and effectiveness of the public health care system. In this paper we extend the model to the case of additional background financial risk. We show that introducing the additional source of risk exerts certain impact on the demand for the supplementary insurance, however its overall effect depends on the patient’s prudence related to a sign of third derivative of the utility function. The paper proposes conditions under which the demand for supplementary insurance increases or decreases, depending on the relation between patient’s prudence with respect to wealth and cross-prudence in health. The results are different from Dudzinski’s article which considered a two-period model.

Keywords: health insurance, risk, prudence.

JEL classification: D81, I13

Introduction

Public health care system in Poland covers most of population and grants access to a wide range of medical services. However, the average time of waiting for health service is 3.1 months, and in some cases it is significantly longer (orthopedics – 9.5 months, stomatology – 7.8 months, angiology – 6.9 months). On average, the waiting time for access to a specialist physician is 2.9 months, in case of orthodontist it is 10.9 months, for endocrinologist it is 10.3 months (WHC
Barometer, 2007). In this situation, private supplementary health insurance becomes an increasingly interesting option for patients. Private health insurance allows for almost immediate access to a primary care physician, and the access to specialist physicians in 5 – 7 days. Consequently, private insurance market has grown significantly in recent years. According to Polish Insurance Association (piu.org.pl), in the third quarter of 2017, the year-over-year growth rate of private policyholders was 23% (28% in 2016) and the total number of policyholders amounted to 2.17 million. Therefore, a theoretical analysis of the demand for private health insurance seems to pose an interesting and important research objective.

The joint analysis of mandatory and private health insurance was introduced in Dudzinski (2015). A mathematical model considers such factors as compulsory healthcare premium, health status of a patient, their wealth, effectiveness of the public healthcare system, and it explains how they impact the demand for private health insurance. The conclusions are rather intuitive: the demand for additional health insurance increases when health status, the effectiveness of public healthcare system or compulsory healthcare premium are on the decrease. Moreover, the additional health insurance is a normal good\(^*\), hence increasing wealth implies increasing demand. For example, between 1990 and 2010 the GDP in Poland increased by 2200\%, whereas health expenditures grew by 3300\% (PIU Report, 2013).

The Dudzinski’s model allows for certain extensions regarding other factors influencing the demand for additional health insurance. Especially interesting is the role of other sources of risk in the decision-making process concerning the purchase of additional health insurance. By definition, the model deals with health risk, but there are other types of independent risk possible. In his work, Dudzinski (2015) considered the effect of a background wealth risk on the demand for health insurance but only in the case of a single health risk management tool. Moreover, Dudzinski (2015) used a two-period model where both risks were separated in time.

This paper considers an individual who is insured in the public healthcare system but due to the limited access to health services they choose a certain level of additional, private health insurance. Apart from health risk, an agent faces another risk of financial type, independent of health risk. The main result of this paper explains how the introduction of financial risk modifies the optimal level of additional health insurance. It is shown that the overall effect depends on the prudence of a patient, mathematically equivalent to convexity of a marginal utility. The concept of prudence was introduced to economic theory by Kimball (1990) to explain and analyze a precautionary savings motive. Since that time, the concept of prudence has been adopted by numerous authors, also in health economics (Hoel 2003, Bui, Crainich, Eeckhoudt 2005). In order to capture the interactions between the two risks, Eeckhoudt, Rey, Schlesinger (2007) extended the concept of prudence to a bivariate case and coined the term „cross-prudence”.

In this paper we prove that the change in demand for additional health insurance depends on which effect prevails - prudence in terms of wealth or cross-prudence with respect to health. A short conclusion is provided in the last section.

1. Additional health insurance – the model without financial risk

Assume that the individual’s preferences are represented by the bivariate utility function \(u(w,h)\) depending on wealth \(w\) and health \(h\). The utility function is assumed to be strictly increasing with respect to each argument, that is

\[
\frac{\partial u}{\partial w} > 0, \quad \frac{\partial u}{\partial h} > 0
\]  

(1)

* Assuming wealth and health are complements.
and wealth-risk averse, as well as health-risk averse; mathematically it is equivalent to convexity with respect to each variable and is represented by the following inequalities:

\[ u_{ww} = \frac{\partial^2 u}{\partial w^2} \leq 0, \quad u_{hh} = \frac{\partial^2 u}{\partial h^2} \leq 0 \]  (2)

Interaction between marginal utility with respect to wealth and health is captured by the condition

\[ u_{hw} > 0 \]  (3)

interpreted as declining marginal utility of consumption with deteriorating health; in this case wealth and health are Edgeworth complements. Theoretical rationale for that assumption is that a health condition makes an individual transform income into utility less efficiently (Roemer 1985). Works of Sloan et al. (1998) and Viscusi and Evans (1990) and Finkelstein et al. (2013) empirically confirm complementarity of wealth and health. On the other hand, works of Lillard and Weiss (1997), Edwards (2008) and Tengstam (2013) suggest that wealth and health are substitutes. That discrepancy between the mentioned findings are explained by various diseases among patients.

The compulsory healthcare premium is denoted by \( p \). The level of additional, private health insurance is denoted by \( e \) – it is a decision variable. Both of them reduce wealth of an individual, which equals

\[ w = w_0 - p - e \]  (4)

where \( w_0 \) denotes initial wealth of a patient.

The size of a health loss of an individual is denoted by \( l(e, \theta) \). It depends also on a state represented by a random variable \( \theta \in [\bar{\theta}, \bar{\theta}] \). An increase in additional health insurance provides higher-quality medical care, and therefore reduces health loss, which means inequality

\[ l_e \leq 0 \]  (5)

It is assumed that the loss function is convex with respect to \( e \):

\[ l_{ee} \geq 0 \]  (6)

The above inequality suggests that as the additional health insurance increases, the health loss decreases, but at a lower rate. It is easily seen that

\[ l(e, \theta) \leq l(0, \theta) \]  (7)

Initial health status denoted by \( h_0 \) is reduced by health loss \( l(e, \theta) \) which, in turn, is reduced due to individual’s investment in additional healthcare \( e \). The individual is also subject to compulsory public health insurance which results in further loss reduction of \( \alpha l(e, \theta) \) where \( \alpha \in [0,1] \) denotes effectivity of a public healthcare system.

The individual’s health status is thus

\[ h = h_0 - l(e, \theta) + \alpha l(e, \theta) = h_0 + (\alpha - 1)l(e, \theta) \]  (8)

The individual’s problem is to choose \( e \) to maximize its expected utility

\[ EU(w, h) = EU(w_0 - p - e, h_0 + (\alpha - 1)l(e, \theta)) \]  (9)

The first-order condition for a maximum of \( EU \) is

\[ \frac{\partial EU}{\partial e} = E[-u_w + (\alpha - 1)l_e u_h] = 0 \]  (10)

The second-order condition for a maximum

\[ \frac{\partial^2 EU}{\partial e^2} = E[u_{ww} - 2u_{wh}(\alpha - 1)l_e u_h + u_h(\alpha - 1)l_{ee}] < 0 \]  (11)
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is satisfied due to our assumptions.

Let \( e^* \) denote an interior solution to the first-order condition (10). It represents the individual’s demand for the additional health insurance.

The model presented above was analyzed by Dudzinski (2015). He showed how changes in individual’s wealth and health as well as in effectiveness of the public health care system affect the demand for additional health insurance. Furthermore, the analysis resulted in finding that the additional health insurance is a normal good, as opposed to wealth insurance, which is inferior (Schlesinger 2000) under decreasing absolute risk aversion. Moreover, the demand for additional health insurance increases when health status, effectiveness of public healthcare system or compulsory healthcare premium decrease. All of the mentioned factors were deterministic, and the only source of risk was health risk. This paper considers introducing another source of risk. That additional risk is of a financial nature, and it concerns wealth of an individual. The aim of the following analysis is to find how the introduction of this new source of financial risk affects the demand for additional health insurance.

2. Additional health insurance – the model with financial risk

Assume that the financial risk is represented by random variable \( \gamma \) taking values over an interval \([\gamma_1, \gamma_2]\). It is assumed that it is a pure risk, i.e. \( E\gamma = 0 \) and is independent of health risk.

An individual’s problem is to choose \( e \) to maximize its expected utility

\[
EU = E_{\theta,\gamma} u(w_0 + \gamma - p - e, h_0 + (\alpha - 1)l(e, \theta))
\]  

(12)

The first-order condition is

\[
\frac{\partial EU}{\partial e} = E_{\theta,\gamma} [-u_w + (\alpha - 1)l_e u_h] = 0
\]  

(13)

The second-order condition is satisfied again due to assumptions on the utility and the loss functions:

\[
\frac{\partial^2 EU}{\partial e^2} = E_{\theta,\gamma} [u_{ww} - 2u_{wh}(\alpha - 1)l_e + u_h(\alpha - 1)l_{ee}] < 0
\]  

(14)

The internal solution to the equation (13) is denoted by \( e^{**} \).

This analysis is intended to compare sizes of \( e^* \) i \( e^{**} \). That will allow to acknowledge the impact of the background financial risk on the demand for the supplementary health insurance. The idea of the proof is based on evaluating sign of \( \frac{\partial EU}{\partial e} \) at the point \( e^* \), the solution to the equation (10).

Recall that

\[
\frac{\partial EU}{\partial e}(e^*) = 0, \quad \frac{\partial EU}{\partial e}(e^{**}) = 0
\]  

(15)

Moreover, the second-order condition (14) is equivalent to the fact that the function \( \frac{\partial EU}{\partial e} \) is on the increase. It follows that

\[
\frac{\partial EU}{\partial e}(e^*) \leq 0 \quad \iff \quad e^* \leq e^{**}
\]  

(16)

\[
\frac{\partial EU}{\partial e}(e^*) \geq 0 \quad \iff \quad e^* \geq e^{**}
\]  

(17)
Observe that the condition (16) may be written as
\[ \frac{\partial E_u}{\partial \varepsilon}(e^\varepsilon) \leq \frac{\partial E_u}{\partial \varepsilon}(e^{\varepsilon'}) \] (18)
which may be expressed as
\[ E_{\theta,\gamma} f(\gamma, \theta) \leq E_{\theta} f(E_{\gamma}, \theta) \] (19)
where
\[ f(\gamma, \theta) = -u_w(w, h) + (\alpha - 1) l_e(e, \theta) u_h(w, h). \] (20)

Inequality (19) was investigated by Finkelshtain, Kella and Scarsini (1999). They found that (19) was equivalent to concavity of \( f(\gamma, \theta) \) with respect to variable \( \gamma \). That in turn is equivalent to inequality
\[ f_{\gamma\gamma} \leq 0 \] (21)
for all values of variable \( \gamma \) in the interval \( [\gamma, \bar{\gamma}] \).

Analogously, the inequality
\[ E_{\theta,\gamma} f(\gamma, \theta) \leq E_{\theta} f(E_{\gamma}, \theta) \] (22)
holds if and only if
\[ f_{\gamma\gamma} \geq 0 \] (23)

Therefore, we are interested in determining the sign of \( f_{\gamma\gamma} \). Subsequent calculations lead to
\[ f_{\gamma}(\gamma, \theta) = -u_{ww}(w, h) + (\alpha - 1) l_e(e, \theta) u_{hw}(w, h) \]
\[ f_{\gamma\gamma}(\gamma, \theta) = -u_{www}(w, h) + (\alpha - 1) l_e(e, \theta) u_{hww}(w, h) \] (24) (25)

The sign of the above expression depends on signs of third-order derivatives of the utility function, namely \( u_{www} \) and \( u_{hww} \).

Having regard to conditions \( \alpha - 1 \leq 0 \) and \( l_e(e, \theta) \leq 0 \), we may conclude what follows:
- if \( u_{www} \geq 0 \) and \( u_{hww} \leq 0 \) then \( f_{\gamma\gamma} \leq 0 \).
- if \( u_{www} \leq 0 \) and \( u_{hww} \geq 0 \) then \( f_{\gamma\gamma} \geq 0 \).

Specific signs of third-order derivatives of the utility function bear some economic meaning. The condition \( u_{hww} \geq 0 \) is equivalent to cross prudence in health (Eeckhoudt et al. 2007). An individual is said to be cross-prudent in health if they prefer a zero-mean risk on their wealth when they are in good rather than poor health. Similarly, inequality \( u_{hww} \leq 0 \) describes an individual who is cross-imprudent in health, meaning an individual who prefers zero-mean risk on their wealth in the case of health deterioration.

The condition \( u_{www} \geq 0 \) is equivalent to prudence in wealth. An individual is said to be prudent in wealth if adding a zero-mean risk to their future wealth raises their savings.

The following proposition summarizes the above considerations.

**Proposition 1.**
3. If an individual is prudent in wealth and is cross-imprudent in health then adding a zero-mean financial risk raises their demand for the supplementary health insurance.
4. If an individual is cross-prudent in health and imprudent in wealth then adding a zero-mean financial risk reduces their demand for the supplementary health insurance.
It is worth pointing out that for determining the sign of $f_{yy}$ it is not necessary for third-order derivatives to have opposite signs. The expression (25) consists in two terms: 

$-u_{w|w_{w}}(w, h)$ and $(\alpha - 1)l_{\theta}(e, \theta)u_{h|w_{w}}(w, h)$. Assuming prudence in wealth and cross-prudence in health, the first term is negative and the second is positive. If the first term outweighs the second term, the whole expression (25) is negative. In that case prudence in wealth dominates over cross-prudence in health. In the opposite case, the sign of (25) is positive. It suggests the following proposition.

**Proposition 2.** Assume that an individual is prudent in wealth and cross-prudent in health. Then

a) if prudence in wealth dominates cross-prudence in health then adding a zero-mean financial risk raises demand for the supplementary health insurance,

b) if cross-prudence in health dominates prudence in wealth then adding a zero-mean financial risk reduces demand for the supplementary health insurance.

**Conclusion**

In his work, Dudziński (2015) analyzed the effect of introducing financial risk on the decision-making process regarding health insurance in various periods. The financial risk in future increases demand for health insurance for the individual who is cross-prudent in health, while the same risk in present time decreased the demand for health insurance for the financially prudent individual. The health risk was assumed to be located in the future. This paper takes into consideration the decision-making process of an individual facing simultaneously both risks, concerning health and wealth. This makes the analysis of a problem more complicated and the results are not as clear as in the case of the time separated risks. In order to determine the direction of a change in the demand for additional health insurance, it is necessary to consider prudence of an individual in respect to wealth and health separately as well as the interactions between them. As the Proposition 2 demonstrates, the demand for additional health insurance may increase or decrease, depending on which effect is stronger. This proves that the concept of prudence is crucial for understanding and explaining the properties and determinants of the demand for health insurance.

**References**


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**WPŁYW RYZYKA FINANSOWEGO NA POPYT NA DODATKOWE UBEZPIECZENIE ZDROWOTNE**

**Streszczenie**

W sytuacji utrudnionej dostępności do świadczeń powszechnej służby zdrowia w Polsce, prywatne, dodatkowe ubezpieczenia zdrowotne są coraz częściej wybieranym sposobem na zapewnienie dostępu do opieki lekarskiej. Rynek prywatnych ubezpieczeń odnotowuje w ostatnich latach dynamiczny wzrost. Interesującym celem badawczym jest więc określenie determinantów popytu na dodatkowe ubezpieczenia zdrowotne za pomocą modelu matematycznego opisującego proces decyzyjny pacjenta ubezpieczonego już w powszechnym systemie opieki zdrowotnej, ale potrzebującego lepszej opieki lekarskiej lub szybszego dostępu do świadczeń zdrowotnych. Model taki został zaprezentowany w pracy Dudzińskiego (2015) a jego analiza wykazała jak popyt na dodatkowe ubezpieczenie zdrowotne zależy od parametrów takich jak wysokość składki na obowiązkowe ubezpieczenie zdrowotne, stan zdrowia pacjenta, jego stan majątkowy oraz efektywność państwowej służby zdrowia. Celem niniejszego artykułu jest zbadanie czy i jak popyt na dodatkowe ubezpieczenie zależy od innych czynników ryzyka, w tym przypadku ryzyka finansowego. Analiza modelu wykazała że wprowadzenie ryzyka dotyczącego dochodów ma wpływ na popyt na ubezpieczenie zdrowotne, przy czym kierunek zmiany jest uzależniony od tzw. przesorności decydenta, wyrażonej odpowiednim znakiem pochodnej cząsteczki trzeciego rzędu
funkcji użyteczności. Popyt może wzrosnąć lub zmalać, jeśli tzw. przezorność fi-
nansowa dominuje nad krzyżową przezornością względem zdrowia, lub odwro-
tnie. Wyniki te różnią się od tych z pracy Dudzińskiego gdzie decyzja oraz źródła
ryzyka były odseparowane w czasie.

Słowa kluczowe: ubezpieczenie zdrowotne, ryzyko, przezorność.

Klasyfikacja JEL: D81, I13

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